e-content for students

B. Sc.(honours) Part 2 paper 4

Subject:Mathematics

Topic:Stable Equilibrium

RRS college mokama

Stability Of Equilibrium

Definition:Stable Equilibrium

Stable equilibrium : *If a body is at rest in such a position that, if slightly disturbed, it would tend to return to its original position, the body is said to be in stable equilibrium.*

Examples : (i) Any body suspended from a point above its centre of gravity.

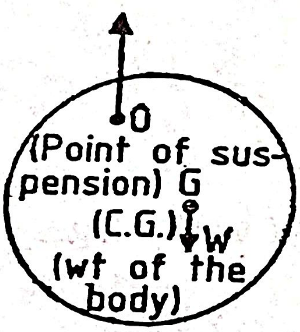


Fig. 1

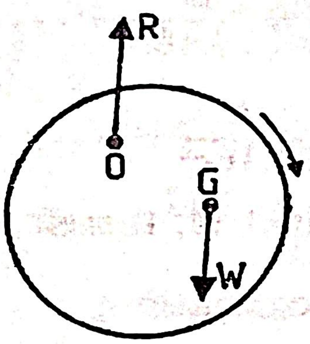


Fig. 2

In the position of equilibrium the centre of gravity *G* of the body will be vertically below the point of suspension, *O* (figure 1).

Now let the body be slightly displaced (figure 2). It will swing for a time and finally return to its original position, as in the displaced position the weight of the body has an unbalanced moment which always tends to bring it back to its original position.

Hence the equilibrium is stable.

(ii) Egg on side.

(iii) A right cone resting on its base.

Theorem: Establish the energy test for stability

Answer : We know that for any dynamical system,

kinetic energy + potential energy = constant.

In the position of equilibrium, kinetic energy = 0.

$\therefore$ Potential energy = constant.

From this we infer that the potential energy is either maximum or minimum.

Let the potential energy be maximum. Now let the system be slightly displaced from the position of maximum potential energy, and then set free. In the new position P.E. will decrease. Consequently the kinetic energy will increase. In other words the kinetic energy will be positive as in the position of equilibrium it was zero. Hence the system moves further away from the position of maximum potential energy. Hence the equilibrium in the position of maximum potential energy is an unstable one.

Again let the potential energy be minimum. Now let the system be slightly displaced from the position of minimum potential energy, and then set free. In the new position the P.E. should increase (as it was already in the position of minimum P.E.). Hence K.E. should decrease. But in the position of minimum P.E., K.E. was zero. Hence K.E. should become negative meaning thereby when released it will have the tendency to move in the direction opposite to the displacement. Hence it will have the tendency to return back to the original position. Thus in this case the equilibrium is stable.

1. z = the height of the C. G. of the body = $f(\theta)$.

2. For equilibrium, $\frac{dz}{d\theta}=0$.

3. For stable equilibrium, $\frac{d^2z}{d\theta^2}>0$.

4. For unstable equilibrium, $\frac{d^2z}{d\theta^2}<0$

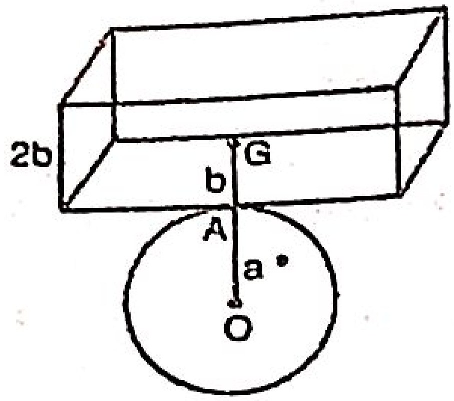
5. If a body of radius r is resting on a body of radius R and h is the height of C. G. of upper body above the point of contact, then for stable equilibrium, $\frac{1}{h}>\frac{1}{r}+\frac{1}{R}$, and for unstable equilibrium, $\frac{1}{h}\leqslant\frac{1}{r}+\frac{1}{R}$.

6. If the lower body of radius R near the point of contact be a plane, then $R=\infty$. $\therefore$ $\frac{1}{R}=0$.

7. If the upper body of radius r near the point of contact be a plane, then $r=\infty$. $\therefore$ $\frac{1}{r}=0$.

8. If a body of radius r resting inside another concave body of radius R and h is the height of C. G. of the body above the point of contact, then for stable equilibrium, $\frac{1}{h}>\frac{1}{r}-\frac{1}{R}$, and for unstable equilibrium, $\frac{1}{h}<\frac{1}{r}-\frac{1}{R}$.

1. *A uniform beam, of thickness 2b, rests symmetrically on a perfectly rough horizontal cylinder of radius a; show that the equilibrium of the beam will be stable or unstable according as b is less or greater than a.*



Solution : Here $r=\infty$, $R=a$, $h=AG=b$.

The equilibrium will be stable or unstable according as

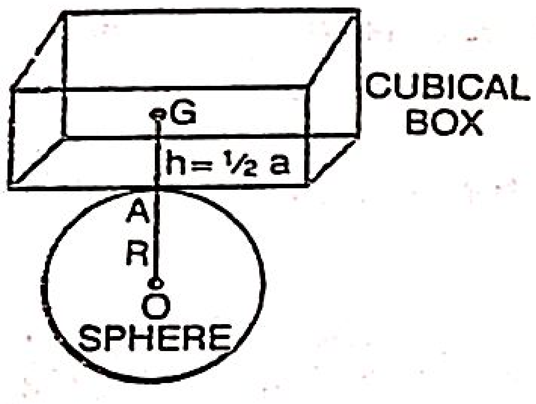
$$\frac{1}{h}> \text{ or } <\frac{1}{r}+\frac{1}{R}$$

i.e. $$\frac{1}{b}> \text{ or } <\frac{1}{\infty}+\frac{1}{a}$$

i.e. $$\frac{1}{b}> \text{ or } <\frac{1}{a};$$

i.e. $$b< \text{ or } >a.$$

2. *A uniform cubical box of edge a is placed on the top of a fixed sphere, the centre of the face of the cube being in contact with the highest point of the sphere. What is the least radius of the sphere for which the equilibrium will be stable ?*



Solution : Here $r=\infty$, $h=\frac{a}{2}$.

Let R be the radius of the sphere. The equilibrium will be stable

provided that $$\frac{1}{h}>\frac{1}{r}+\frac{1}{R};$$

i.e. $$\frac{2}{a}>\frac{1}{\infty}+\frac{1}{R};$$

i.e. $\frac{2}{a}>\frac{1}{R}$; i.e. $\frac{a}{2}<R$; i.e. $R>\frac{a}{2}$.

Hence the least value of R is $\frac{a}{2}$.